

Alex's Bach-ventures in Numberland...

I have a theory that Bach uses 6 numbers to govern the architecture of his compositions: 3 are religiously significant; and 3 are personally significant.

Religiously significant: **3** (divine perfection/the Trinity), **4** (earthly perfection, e.g. Spring, Summer, Autumn, Winter, North, South, East, West, Earth, Air, Fire, Water, etc.), **10** (used for completion in the Bible, e.g. ten commandments).

Bach's personally significant numbers arise from using Gematria, where numbers are matched to letters of the alphabet (there was no J in the Latin alphabet, and U/V were the same letter). We think Bach used this based on others before him, so:

A	B	C	D	E	F	G	H	I	K	L	M	N	O	P	Q	R	S	T	V	W	X	Y	Z
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

$B + A + C + H = 2 + 1 + 3 + 8 = 14$ (hence first fugue subject in the Well-Tempered Clavier has 14 notes, etc.)

$B \times A \times C \times H = 2 \times 1 \times 3 \times 8 = 48$ (hence 48 preludes and fugues in the Well-Tempered Clavier, etc.)

Soli Deo Gloria (SDG) translates to 'To God Alone be Glory' = S D G = $18 + 4 + 7 = 29$

So Bach uses three religiously significant numbers (**3**, **4** and **10**), and three personally significant numbers (**14**, **29** and **48**).

Bach regularly also makes use digit sums, as was common in medieval numerology. Taking the three personally significant numbers:

$14 + 29 + 48 = 1 + 4 + 2 + 9 + 4 + 8 = 28$ (**14** + **14**), or $(B+A+C+H) + (B+A+C+H)$ – helpful.

For the religiously significant numbers:

$3 + 4 + 10 = 17$; the product of **14** $(B+A+C+H)$ + **3** (Divine Perfection/the Trinity)

And $3 + 4 + 1 + 0 = 8$, the product of **14** – **3** – **3** $(B+A+C+H)$ – (Divine Perfection/the Trinity) – (Divine Perfection/the Trinity)

So, as a teaser for just how clever the music is for the John passion, I thought I'd share with you how this explains one of the most famous introductions in the history of music...

Movement I (Chorus)

The introduction of Bach's St John Passion lasts exactly 18 bars and one beat (the choir begin 'Herr', meaning 'Lord', on the final beat of the introduction).

$18 = 14$ $(B+A+C+H)$ + **4** (Earthly Perfection; i.e. the best that he as a human can achieve)

With 4 beats in each bar, and one beat at the end: $[(14 + 4) \times 4] + 1 = 73$ beats. So what? But remember digit sums were used extensively in medieval numerology, and can sometimes reveal even more:

$73 = 7 + 3 = 10$ (Biblical completion)

So that's quite neat. Next we need to look at the four layers in the music: the melody shared across the two oboes, the accompaniment, and the bass line.

Oboe I

Oboe II

Accompaniment (the churning thirds)

Bass line

The work opens in G minor. Assigning a number to each degree of the scale (ignoring accidentals, so F and F# when they are used are both a version of the 7th degree of the scale):

G	A	B	C	D	E	F
1	2	3	4	5	6	7

Oboe I's melody begins Eb, C, F#, etc., which would be $6 + 4 + 7$, etc.

Adding up the degrees of the scale used for Oboe I's melody across the whole introduction gives 107 (a prime number). Writing the sum of 107 out, and adding the digits:

$$1 \times 107 = 1 + 1 + 0 + 7 = 9 = 3 \times 3.$$

Oboe II's melody begins D, G, C, Bb, etc. which would be 5 + 1 + 4 + 3, etc.

Adding up the degrees of the scale used for Oboe II's melody across the introduction gives 146.

146 in factors becomes:

$$1 \times 146; \text{ or} \\ 2 \times 73$$

Adding the factor digits:

$$1 + 1 + 4 + 6 = 12 = 3 \text{ (Divine Perfection)} \times 4 \text{ (Earthly Perfection)} \\ 2 + 7 + 3 = 12 = 3 \text{ (Divine Perfection)} \times 4 \text{ (Earthly Perfection)}$$

Convenient. But why these two numbers (107 and 146)?

Because they add to 253 which, when you add the digits (2 + 5 + 3) gives **10** (completion – i.e. adding the scale degrees used in the melodic lines over the whole introduction).

Looking at the factors of 253:

$$1 \times 253; \text{ or} \\ 11 \times 23$$

Adding the factor digits:

$$1 + 2 + 5 + 3 = 11 = 14 \text{ (B+A+C+H)} - 3 \text{ (Divine Perfection/the Trinity)} \\ 1 + 1 + 2 + 3 = 7 = 3 \text{ (Divine Perfection/the Trinity)} + 4 \text{ (Earthly Perfection)}$$

So that explains the two Oboe parts and why they play those notes!

Then we look at the churning accompaniment in thirds. The one-bar idea begins in bar 1 and then develops.

The notes of the thirds in the first bar of the accompaniment are Bb/D, A/C, Bb/D, C/Eb, and this is repeated **4** times in bar 1. Applying the numbers again:

G	A	B	C	D	E	F
1	2	3	4	5	6	7

3 + 5 + 2 + 4 + 3 + 5 + 4 + 6 = 32, which can conveniently be made from:

$$14 \text{ (B+A+C+H)} + 4 \text{ (Earthly Perfection)} + 14 \text{ (B+A+C+H)}$$

It gets better... That figure is repeated 4 times, so $4 \times 32 = 128$.

Note that $4 \times 32 = 4 + 3 + 2 = 9 = 3 \text{ (Divine Perfection)} \times 3 \text{ (Divine Perfection)}$

And, taking the whole sum, $4 \times 32 = 128$ and adding all the digits:

$$4 + 3 + 2 + 1 + 2 + 8 = 20 = 10 \text{ (Biblical completion)} + 10 \text{ (Biblical completion)}$$

It gets better still, because Bach chose 128 for another reason:

Looking at the factors of 128:

$$1 \times 128; \\ 2 \times 64; \\ 4 \times 32; \text{ or} \\ 8 \times 16$$

Now we add the digits:

$$1 + 1 + 2 + 8 = \underline{12}$$

$$2 + 6 + 4 = \underline{12}$$

$$4 + 3 + 2 = \underline{9}$$

$$8 + 1 + 6 = \underline{15}$$

None of these underlined numbers are helpful until you realise that $12 + 12 + 9 + 15 = 48$, which is $2 \times 1 \times 3 \times 8$, or:

$$B \times A \times C \times H$$

So that's the one bar accompaniment idea explained, which is then varied throughout the introduction.

The final layer is the bass line. Again, looking at the order of notes in the bass line of the whole introduction (bearing in mind the whole of the first page is a G):

G (1st page), D, G, C, F, Bb, E, Eb, D, C#, D, G

Using the table of scale degrees:

G	A	B	C	D	E	F
1	2	3	4	5	6	7

That becomes:

$$1 + 5 + 1 + 4 + 7 + 3 + 6 + 6 + 5 + 4 + 5 + 1$$

Adding that gives $48 = 2 \times 1 \times 3 \times 8$ or, once again, $B \times A \times C \times H$.

So Bach writes his name into the bass line, in a nice parallel to the idea of him being lowly on earth, offering up the counterpoint above to God.

AND THAT'S JUST THE INTRODUCTION OF THE FIRST MOVEMENT! You can listen to it [here](#). Enjoy!

It's easy to see why Bach became interested in Gematria and numerology. Gematria existed in biblical times, and numerology was quite significant in medieval thought, since they believed it provided a link between the earthly and divine.

We think Bach first got fascinated by it because of the symmetry achieved with his name.

$$B + A + C + H = 2 + 1 + 3 + 8 = 14$$

$$J + S + B + A + C + H = 9 + 18 + 2 + 1 + 3 + 8 = 41 \text{ (the mirror image of 14)}$$

Perhaps this explains his fondness for canons, fugues, crab canons and other rather mathematical approaches to composition.

It also works if we use the reverse operation of Gematria. Bach's birth date was 21 March 1685.

In numbers, that's 21 03 85

$$2 + 1 + 3 + 8 + 5, \text{ in letters, becomes } B + A + C + H + (E)$$

What are the chances of someone's date of birth spelling their surname in gematria, and the name being able to be written in symmetrical numbers? We can go even further...

He died on 28 July 1750:

28 07 1750 in a slightly different format, because...

$$2 + 8 + 0 + 7 + 1 + 7 + 5 + 0 = 30 = 3 \text{ (Divine Perfection)} \times 10 \text{ (Completion)}$$

Which leads to the question of whether Bach might have chosen to die on that day because of the numerology...

For those wondering, assigning the numbers of his death day to letters gives:

B + H + G + A + G + E, which is an anagram of BEG. GAH! (maybe he meant 'bugger?'). Too far, perhaps...

The Text of Movement I

Bach deliberately repeats words to create an overall numerological scheme. The tallies below include the da capo.

3 = Divine Perfection/the Trinity

4 = Earthly Perfection

10 = Biblical Completion

14 = B+A+C+H

29 = S+D+G (Soli Deo Gloria, or 'To God Alone Be Glory')

48 = BxAxCxH

Text	Translation	Repetitions				Total	Numerological Explanation
		S	A	T	B		
Herr	Lord	26	38	36	26	126	$1 + 2 + 6 = 9$ (3×3); $126 = 14 \times 3 \times 3$
unser	our	20	18	24	26	88	$88 + 88 = 176$; $1 + 7 + 6 = 14$
Herrscher	Ruler	20	18	24	26	88	Adding digits of factor pairs = 66; $6+6 = 12$ (3×4)
dessen	whose	10	10	10	10	40	10 = Biblical completion; $40 + 40 = 80$;
Ruhm	Praise	10	10	10	10	40	Adding digits of factor pairs of 80 = $42 = 14 \times 3$
In allen Landen	In all Lands	12	12	12	14	50	$12 = 3 \times 4$; $14 = B+A+C+H$ Adding digits of factor pairs of 50 = 21 $21 = 14 + 3 + 4$ or $(4 + 3) \times 3$
herrlich	glorious	14	16	14	12	56	$56 = 14 \times 4$; adding digits of factor pairs of 56 = 48
ist	is	6	6	6	6	24	$56 + 24 = 80$; adding digits of factor pairs of 80 = 42 $42 = 14 \times 3$; 24 is $14 + 10$ (and half of 48).
Zeig uns durch deine Passion	Show us through your Passion	2	2	2	3	9	$9 = 3 \times 3$
Daß du, der wahre Gottessohn	That you, the true Son of God	3	3	3	2	11	$11 = 14 - 3$
Zu aller Zeit Auch in der größten Niedrigkeit	At all Times, even in the greatest abasement/humiliation	2	2	2	2	8	$8 = 14 - 3 - 3$
Verherrlicht	Glorified	9	10	11	12	42	$42 = 3 \times 14$
worden	been	8	7	8	8	31	$31 = 14 + 3 + 14$
bist	have	8	7	8	7		$30 = 3 \times 10$ (divine perfection x Biblical completion)

The Structure of Movement I

Intro, then A, B and da capo A

Intro = 73 beats (7×10) + 3

To end of A section (1st time) = 229 beats

229 = a prime number, so 1×229

Adding the digits: $1 + 2 + 2 + 9 = 14$

Whole movement = 377 beats

$337 = 1 \times 337$ or 13×29

Adding digits:

$1 + 3 + 3 + 7 = 14$; or

$1 + 3 + 2 + 9 = 15$. OH NO!

But:

$14 + 15 = 29$ (SDG, or Soli Deo Gloria; 'to God alone be Glory')

Proves that the final rests in b.95 are notational convenience, and not necessarily to be followed...

Anyway, a little hint as to how clever this music is!